Math 114 **ACTIVITY 7:** Applications of the normal distribution

Why

The normal family of distributions is one of the best-known and most-used families of distributions. It will form the basis for the inference methods studied in this course. To make use of the normal variables, it is necessary to be familiar with the computation both “forward” (probabilities for events described on the normal curve) and “backward” (percentiles, quartiles, cutoffs and values that produce desired probabilities).

LEARNING OBJECTIVES

1. Be able to find normal distribution probabilities using standardization and the table of standard normal probabilities
2. Be able to find cutoff values for specified portion of a normally distributed variable (to 5%, middle 90%, etc.)
3. Become comfortable with the idea of probability represented by area under a density curve.

CITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Success in involving all members of the team in the conversation.
3. Success in completing the exercises.

RESOURCES

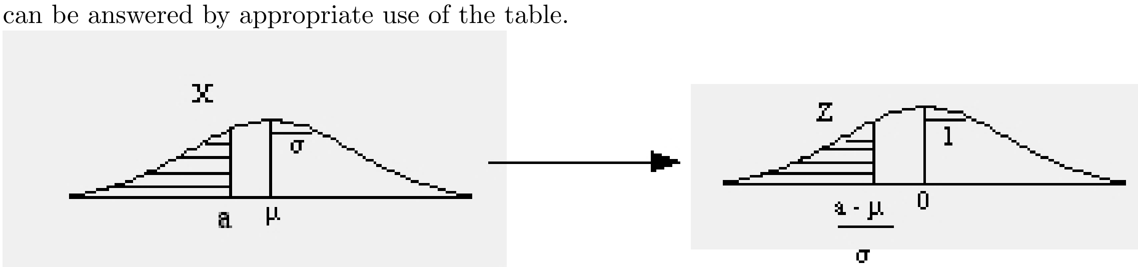
1. Your text, section 7.3 and the standard normal probability table (Table V) pages A-11 & A-12
2. The team role desk markers (handed out in class for use during the semester)
3. Your class notes from Monday and Wednesday
4. Your calculator
5. 40 minutes

PLAN

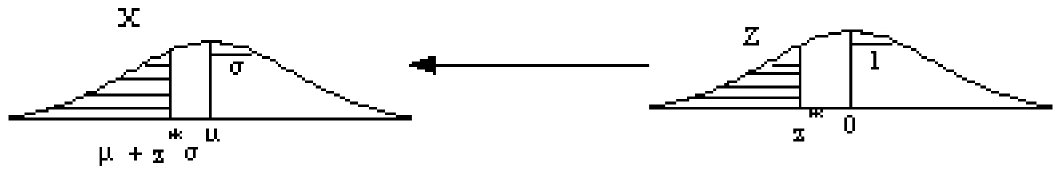
1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the exercises given here - be sure everyone understands all results & procedures(30 minutes)
3. Assess the team’s work and roles performances and prepare the Reflector’s and Recorder’s reports including team grade (5 minutes).

DISCUSSION

If X is a normally distributed variable, with mean μ and standard deviation σ then the *standardized variable* $Z = \dfrac{X-\mu}{\sigma}$ is a *standard* normal variable, and information on its probability distribution is given by the standard normal table. Thus, for any number a, $P(X < a)$ is equal to $P\left(Z < \dfrac{a-\mu}{\sigma}\right)$—which can be obtained from the table. All other probability questions translate similarly to Z-probability questions, which can be answered by appropriate use of the table.



“Reverse” or “cutoff” questions can also be answered by standardization and use of the table – but the Z question is also a “reverse” question. First we find the Z-interval that gives the desired probability, then “de-standeardize” using the formula $X = \mu + Z\sigma$. [For example, if we want the value of X that cuts of the top 5% of the values, it will be obtained from $Z_{.05}$ (the value of Z that cuts off the top 5% of the Z values) by *de-standardizing*: $X = \mu + Z\sigma]$

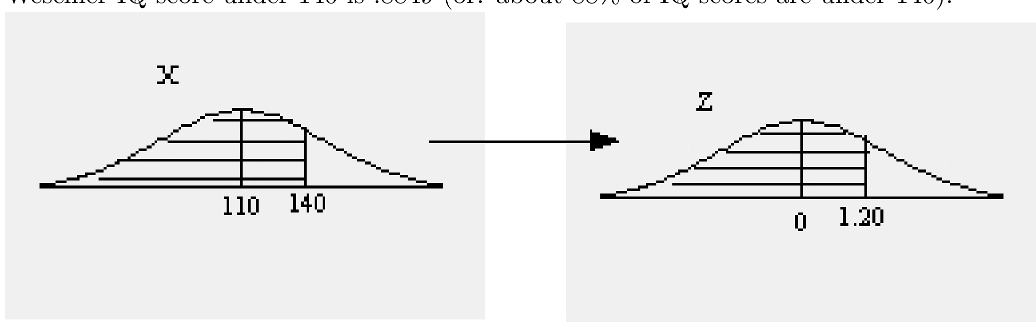


MODEL

Scores on the Weschler Adult Intelligence Scale (one of several standard "IQ" tests)for the 20 to 34 age group are approximately normally distributed, with $\mu = 110$ and $\sigma = 25$. In this example, we let X = Weschler IQ score of a randomly selected person in the 20 to 34 age group.

1. Some probability examples – we standardize the X-values ($Z = \frac{X-\mu}{\sigma}$) and use the Z - table from outside to inside:

   (a) The probability of getting a person with a score under 140 is $P(X < 140) = P(Z < \frac{140-110}{25}) = P(Z < 1.20)$ Using the Z- table (Table V), we find this probability is .8849. The probability of getting a person with a Weschler IQ score under 140 is .8849 (or: about 88% of IQ scores are under 140).

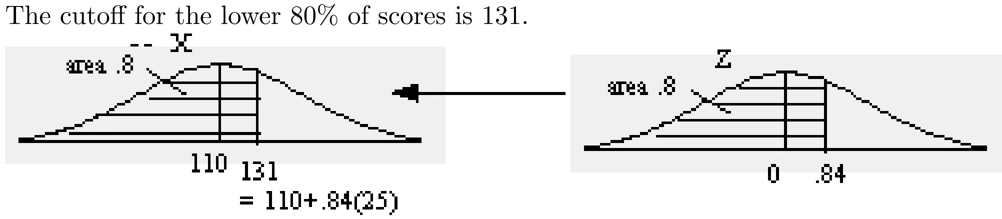
   

   (b) The probability of getting a person with a score over 138 is $P(X > 138) = P(Z > \frac{138-110}{25}) = P(Z > 1.12) = 1 - P(Z < 1.12) = 1 - .8686$ [Switch to "less than" because that's what's in the table— in fact probabilities involving "$Z >$ " *always* require subtraction of probabilities.]

   (c) The probability of getting a person with a score between 100 and 150 is $P(100 < X < 150) = P(\frac{100-110}{25} < Z < \frac{150-110}{25}) = P(-.4 < Z < 1.6) = P(Z < 1.6) - P(Z < -.4) = .9452 - .3446 = .6006$ About 60% of the IQ scores are between 100 and 150. [Note we wind up with a subtraction because we had "$Z > -.4$" .]

2. Some "cutoff" examples – we find the Z-value (using the table in reverse – from inside to outside) and then "de-standardize" ($X = \mu + Z\sigma$)

   (a) To find the 80-th percentile (Value a with 80% of the IQ scores below a):
   Find z' with $P(Z < z') = .80$ (from table: $z' = .84$ (closest probability in table is .7995)so $a = 110 + (.84)(25) = 131$.
   The cutoff for the lower 80% of scores is 131.

   (b) The range for the middle 80% of scores:
   Find z' for which $P(-z' < Z < z') = .80$ This requires that $P(Z < -z') = .10$, so $-z' = -1.28$ and (just to check) $P(Z < z') = .90$ so $z' = 1.28$
   Then (using "X_L" for "the left-hand X-value" and "X_R" for "the right-hand X-value") the range for X -values is from $X_L = 110 - 1.28(25) = 78$ to $X_R = 110 + 1.28(25) = 142$
   The interval for the middle 80% of Weschler IQ scores is from 78 to 142.

EXERCISE In calculating, write the probabilities being found using the $P(X < \ )$, $P(Z < \ )$ form(including $P(X > \ )$, $P(\ < Z < \ )$, etc.); be sure to state the answer to the question asked (using words and symbols). It would be helpful to draw and shade a normal curve, indicating the area to be found.

1. The length of human pregnancies varies according to a distribution which is approximately normal with mean 266 days and standard deviation 16 days.
   (a) What percent of pregnancies last less than 240 days (about 8 months)?
   (b) What percent of pregnancies last more than 270 days (about 9 months)?
   (c) What percent of pregnancies last between 240 days and 270 days (about 8 to 9 months)?
   (d) What is the cutoff separating the longest 20% of the pregnancies from the other 80%?
   (e) What is the cutoff separating the shortest 30% of the pregnancies from the other 70%?
   (f) What interval contains the middle 90% of pregnancy lengths?
2. The heights of adult American men are approximately normally distributed, with $\mu = 69$ inches, $\sigma = 2.5$ inches.
   (a) What percent of men are at least 6 feet (72 inches) tall?
   (b) What percent of men are between 5 ft (60 inches) and 6 feet (72 inches) tall?
   (c) How tall must a man be to be in the tallest 10% of all adult men?
   (d) What interval gives the middle 90% of all men's heights?

READING ASSIGNMENT (in preparation for next class)
Section 7.5 on the normal approximation to the binomial distribution

SKILL EXERCISES:(hand in - individually - at next class meeting) Sullivan p.354 #5–8, 14–15, 17–18, 24–26