Math 114 **ACTIVITY 9:** Estimation (confidence intervals) for a population proportion p

Why

We used the sampling distribution for $\bar{x}$ to develop a method for approximating the mean of a population with some control on the precision (error allowance) and confidence (chance of actually capturing the population mean). This is our basic tool for estimation with quantitative variables (for which the mean makes sense). In working with qualitative variables, all we can describe is the *proportion* of the population that fits in a particular category. Thus we want to use the sampling distribution of sample proportions to develop a way to estimate a population proportion

LEARNING OBJECTIVES

1. Be able to calculate and interpret a confidence interval for a population proportion.
2. Be able to tell when the normal model is appropriate for sample proportions
3. Be able to calculate the sample size necessary to estimate a proportion with a given level of confidence and degree of precision.
4. Understand the effects of sample size, confidence level, and required precision (error bound) on the size of a confidence interval.

CRITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Success in involving all members of the team in the conversation.
3. Success in completing the exercises.

RESOURCES

1. Your text, section 9.3 (and section 8.2 as background)) and Table VI (bottom row)p.A-13 [critical values for t - use bottom row for Z]
2. The team role desk markers (handed out in class for use during the semester)
3. Your class notes from Monday and Wednesday
4. Your calculator
5. 40 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the exercises given here - be sure everyone understands all results & procedures(30 minutes)
3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade (5 minutes).

DISCUSSION

Estimation of a population proportion p works similarly to estimation of a population mean μ. Our point estimate (single value estimate) for p is $\hat{p}$. We need the sample to be a small proportion of the population ($n \leq .05N$) in order for our sampling to be binomial – so that we know $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ – but large enough by itself ($np(1-p) \geq 10$) that the distribution of $\hat{p}$ is approximately normal. When we are estimating, we don't know the value of p (why would we estimate if we knew?), so we have to approximate $\sigma_{\hat{p}}$ by $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, and we use an approximate test for sample size: we test for $n\hat{p}(1-\hat{p}) \geq 10$

When all our conditions are right, we estimate the population proportion p, with confidence $(1-\alpha)$ by

$\hat{p} - E$ to $\hat{p} + E$ with $E = Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Notice this is a Z, not a t, because the standard deviation is tied to the proportion – it's not a *separate and independent* source of variation.

Based on this formula, we also have a formula for the minimum sample size needed to obtain confidence $(1-\alpha)$ and keep the error estimate down to E or less. There are two situations:

In general (worst-case calculation − largest sample required if $p = .5$) $n \geq .25 \times \left(\frac{Z_{\frac{\alpha}{2}}}{E}\right)^2$

If we have a preliminary estimate (maybe from a preliminary sample ? maybe from other studies?) $p*$ for p, then we can use the formula $n \geq (p*)(1-p*) \times \left(\frac{Z_{\frac{\alpha}{2}}}{E}\right)^2$ [this gives a smaller value for n, if $p*$ is not .5]

MODEL

A mail-order company is studying the process of filling customer orders. According to company standards, an order is shipped on time if it is sent within 3 working days of the time it is received. The director of quality control selects a simple random sample of 100 out of the 5000 orders received in the past month, and finds that 86 of these orders were shipped on time. Since the sample is 2% (less than 5%) of the population, we consider our sampling to be binomial.

1. To estimate the proportion of orders filled on time, with confidence 95%:
   Since $\hat{p} = \frac{86}{100}$ and $100(.86)(.14) = 12.04 \geq 10$ we can use the normal approximation to the binomial to find our error allowance. $E = Z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.960\sqrt{\frac{.86(.14)}{100}} =, 068$
   With 95% confidence, we say that between 79.2% and 92.8% of the orders last month were shipped on time.

2. To estimate the proportion of orders filled on time, with confidence 90%:
   Since $\hat{p} = \frac{86}{100}$ and $100(.86)(.14) = 12.04 \geq 10$ we can use the normal approximation to the binomial to find our error allowance. $E = Z_{.05}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.645\sqrt{\frac{.86(.14)}{100}} = .057$
   With 90% confidence, we say that between 80.3% and 91.7% of the orders last month were shipped on time.

3. If another person takes a different sample of 100 orders (from the same 5000 orders last month) and finds 91 shipped on time her 95% confidence estimate of the proportion shipped on time will require an error allowance $1.960\sqrt{\frac{.91(.09)}{100}} = .056$ Her result will be
   " With 95% confidence, we estimate that 85.4 to 96.6% of orders were shipped on time."
   Two things to notice:
   First, the error allowance is different, because our estimate of the standard deviation of $\hat{p}$ depends on the proportion—it varies by the sample but not *independently* of the value of $\hat{p}$. [so we can use Z instead of t].
   Second, the two 95% confidence intervals are not the same—but they *are consistent*. Since they overlap, it is quite possible that both have succeeded in capturing the actual (population—all 5000 orders) proportion.

4. If we wanted to estimate the proportion shipped on time with 95% confidence but error no more than .05, we could take advantage of the first study to say the sample size needed would be at least $n \geq (.86)(.14)\left(\frac{1.960}{.05}\right)^2 = 185.01\ldots$, so we would need to study at least 186 orders.
   If we did not have the information from the preliminary (.86) estimate, we would use the "worst-case" calculation and say the sample size should be at least $n \geq .25\left(\frac{1.960}{.05}\right)^2 = 384.1\ldots$ - we would study at least 385 orders. [More information allows better planning]

EXERCISES

1. Until recently, the fatality rate for herpes simplex encephalitis (a viral brain infection) has been 70%. In a preliminary test with a new drug, 50 cases were treated and there were 14 deaths.
   (a) What is the 90% confidence estimate of the fatality rate for people treated with the new drug? [Population size is not an issue here, since this is an experiment, rather than an observational study, but the question of normality is still important].
   (b) Based on your answer to a.), would you say we have evidence that the drug reduces the fatality rate? [We will see more formal methods for dealing with this question beginning next week]
   (c) What is the 95% estimate of the fatality rate with the drug? How does this interval compare to the 90% confidence interval?
   (d) If the study had been four times as large (200 cases) and produced the same fatality rate (28%), what would we have obtained for a 90% confidence interval on the fatality rate? How does this compare to your result in (a.)?
   (e) The company developing the drug needs to conduct another study (for FDA approval) and wishes to estimate the fatality rate within .05 with confidence 95%. How many cases will they need in this study (Base your calculations on the results of this first test).

2. Two studies of the "stop smoking" program we considered before have been conducted. In the first study, 150 participants were studied and 72 were still non-smokers (two years later) – giving a 48% success rate; in the other, 270 participants were studied and 143 were still non-smokers – giving a 53% success rate .

   (a) Give the 95% confidence interval estimates for the proportion of non-smokers among participants in the program.

   (b) Are the two estimates in conflict, or is it possible that both are correct (= include the true proportion)?

   (c) Does either of these studies indicate that the program works for more than half the participants (or that it fails for more than half)?

READING ASSIGNMENT (in preparation for next class)
Section 10.1 - The language of hypothesis testing [You should also read over section 9.4 summarizing confidence intervals for means and proportions]

SKILL EXERCISES:(hand in - individually - at next class meeting) Sullivan **p. 441** #8-9, 15, 19 - 20, 25 and **p. 450** (chapter review exercises) #13 - 15