Math 251/Buad427 **ACTIVITY 2:** Sensitivity Analysis in linear programming

Why

In any linear programming problem there are parameters that are estimated or that are subject to change. The linear programming model allows us to readily asses the effects of many of the most critical changes. In particular, we can look a the effects (on the optimal solution and/or the optimal value of the objective) of a change in 1.) a coefficient of the objective function or 2.) the right-hand side of a constraint.

LEARNING OBJECTIVES

1. Work as a team, using the team roles.
2. Know how to determine whether changes in objective function coefficients will change the optimal solution of a linear programming problem
3. Know how to determine the effects on the objective of small changes in a constraint.
4. Know how determine what is a "small" change in a constraint.

CITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Understanding of the material by all team members
3. Success in completing the exercises.

RESOURCES

1. Your class notes on linear programming and the notes on "Sensitivity Analysis" [handout from Wednesday]
2. Your text section 2.4
3. Microsoft Excel and the workbook TOYCOproduction.xls available in the Course Documents section of the Blackboard site (Linear Programming Notes folder)
4. 50 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3
2. Work through the exercises given here - be sure everyone understands all results
3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade.
4. Be prepared to discuss your results

EXERCISES

We will work with the problem given below [TOYCO company production]. The LP model is given below, and the spreadsheet model is prepared (but not solved] in a model available as TOYCOproduction.xls in the Course Documents area [Linear Programming Documents folder] of the Blackboard site for the course.

1. Read through the situation (next page) and the LP model, open the spreadsheet model, and use the solver to find the optimal solution. Save the Sensitivity analysis.
2. Use the solution and the sensitivity analysis to answer the following questions.
   (a) What is the optimal solution? What is the value of the solution?
   (b) Which constraints are binding constraints? Give the slack or surplus for each of the others
   (c) Why is the reduced cost for the number of trucks not equal to 0?
   (d) How low would the profit per car have to go to change the optimal profit mix? How high?
   (e) If the company decided that it must make some trucks, what effect would that have on the daily profit (increase? decrease? how much per truck?)? How can you tell?

(f) If the profit per train decreased to \$4, what would be the effect on the optimal production plan? what would be the effect on profit?

(g) What effect would there be on the profit if the time available for Operation 3 increased to 450 minutes? Why does it make sense that the shadow price for available Operation 3 time is 0? How low would the available time on Operation 3 have to drop to change this shadow price?

(h) Suppose the company could extend the available time for Operation 1 by 20 minutes per day at a cost of \$.50 (per minute). Use the shadow price to explain why this would be worthwhile for the company. Would this change the optimal production mix?

(i) If problems with machines reduce the available daily time for Operation 2 to 400 minutes, what affect will this have on profit (be specific — a dollar amount)? Why can't we predict (without re-writing and re-solving the problem) the effect of a decrease to 290 minutes?

(j) If the contract for toy cars changed to require 90 toy cars per day, how would this affect the optimal solution? The value?

READING ASSIGNMENT (in preparation for next class meeting)

Read Section 2.7 (special solution situations)

SKILL EXERCISES:(hand in - individually - at next class meeting)

Text p. 101: #9, 26 (for now, don't worry about whole number of pies)

The situation – TOYCO Company production (modified from Taha, Operations Research 7th ed, Prentice-Hall, 2003 p.133)

TOYCO assembles three types of toys: trucks, cars and trains, using three operations. Operation 1 can be carried out for a maximum of 430 minutes per day, Operation 2 for 460 minutes, Operation 3 for 420 minutes. Assembling a toy truck requires 1 minute in Operation 1, 3 minutes in operation 2, and 1 minute in Operation 3. Assembling a toy car uses 2 minutes in operation 1, does not use Operation 2, and requires 3 minutes in Operation 3. Assembling a toy trrain uses one minute in Operation 1, 2 minutes in Operration 2, and no tme in Operation 3. The company makes \$3 per toy truck, \$2 per toy car and \$5 per toy train, and can sell all the toys made. Because of an existing contract, TOYCO must make at least 80 toy cars per day. The company wishes to decide on a daily production mix to maximize profit.

The Model:

Set variables
$X1 =$ number of toy trucks made per day
$X2 =$ number of toy cars made per day
$X3 =$ number of toy trains made per day
maximize daily profit $z = 3X1 + 2X2 + 5X3$
subject to:
$X1 + 2X2 + X3 \leq 430$ (operation 1 time - minutes)
$3X1 + 2X3 \leq 460$(operation 2 time - minutes)
$X1 + 3X2 \leq 420$(operation 3 time - minutes)
$X2 \geq 80$ (Contract)
$X1, X2, X3 \geq 0$ (nonnegativity)