

## Why

Our purpose in studying sets is to use them for understanding and analyzing descriptions of objects, actions, results. Most of the important ideas can be visualized in terms of Venn diagrams, which give a visual representation of sets and combinations of sets as areas in a plane (with all the objects that fit a particular description grouped together). When we have several sets (corresponding to several descriptions) the combinations or unions, intersections, and complements can be seen and the connections between them analyzed more easily.

## LEARNING OBJECTIVES

1. Be able to work effectively as a team, using the team roles
2. From a description of combination of sets, be able to identify the corresponding region in a Venn diagram.
3. Be able to write the set description of a region marked on a Venn diagram.
4. Be able to write and interpret the set notation for a combination of descriptions.

## CITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Success in involving all members of the team in the conversation.
3. Success in constructing the diagrams to represent different combinations of sets.
4. Success in interpreting Venn diagrams and writing set notation.

## RESOURCES

1. The course syllabus
2. The team role desk markers (handed out in class for use during the semester)
3. Your text - section 1.2
4. 50 minutes

## PLAN

1. Read the discussion before class
2. Select roles, if you have not already done so, and decide how you will carry out steps 3 to 5 (5 minutes)
3. Work through the exercises given here - be sure everyone understands all results & procedures(40 minutes)
4. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade (5 minutes).

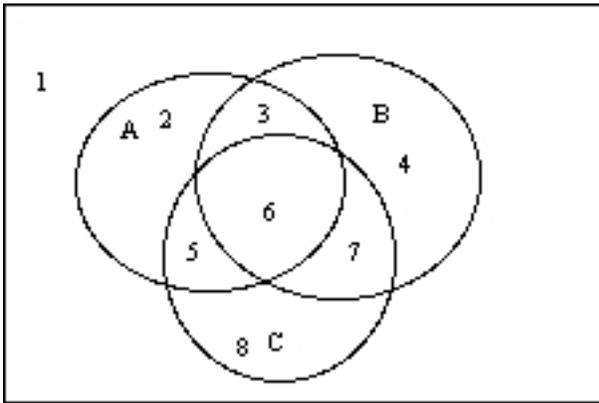
## DISCUSSION

This is a brief summary of the relevant part of the reading you have done for today. There are examples in section 1.2 of your text.

We represent a set in a diagram by drawing a boundary - the objects in the set are inside the boundary, the others outside (We usually label the region -  $A$ ,  $B$ , etc. - that represents the inside); we put a "frame" representing the boundary of the universal set - everything that can be in a set is inside this boundary.

If we do have any special information about the sets, we draw them in a general position - as if they overlap, but not completely (as if some, but not all, members of  $A$  are members of  $B$ , etc.). Then the *intersection* of two sets is represented by the area where their regions overlap - the points that are in both sets. The *union* is represented by putting the regions together (extending one by the other) - the points that are in at least one of the sets. The *complement* of a set is represented by all the points that are *not* in the set (the "outside").

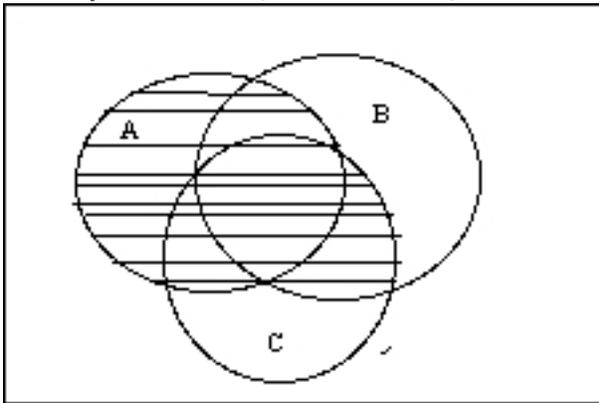
For two sets, the drawing give us four regions described by the sets (Not in either set, in  $A$  but not in  $B$ , in both, in  $B$  but not in  $A$ ); for three sets the universe is partitioned into eight pieces.



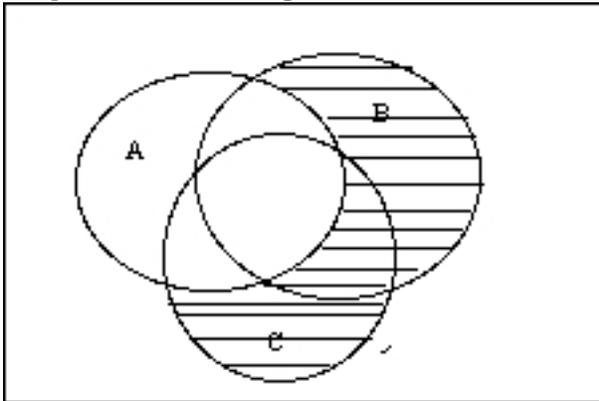
Each piece matches a three-way description - for example the section labeled 7, here is inside B and inside C but outside A - so it represents  $A' \cap (B \cap C)$

To mark the region corresponding to a set description - such as " $(A \cup B) \cap C$ " we look at each region and say "Do these points fit the description?" . If they do, we mark them; if not, we don't. For example, to mark the area for " $A \cup (B \cap C)$ " we say: Points in 1? - no (not in A or in B or in C); Points of 2? - yes (in A - so in A or ....) In fact, all the points of sections 2, 3, 5, 6 are included, because they are in A. Points in 4? No - not in A and not in *both* B and C. Points in 7? Yes - not in A, but they are in both B and C.

To avoid the region-by-region checking (which is always legitimate) we could look at the description and say "It's a union - *all* of A is included, and we add any parts of  $B \cap C$  that aren't already in A. Either way, our area is [uneven spacing has no meaning except lack of skill by artist]:



To read a diagram, we say "What is included, what is excluded?" Including a region is usually done with a union or by simply starting with the (big) set. Excluding is usually done by taking an intersection with a complement: In this diagram:



everything marked is in B or in C, but is outside A so the region is  $A' \cap (B \cup C)$ .

We could also say the marked area is "all of B that's not in A plus (union) all of C that's no in A" and say (correctly) that the region  $(A' \cap B) \cup (A' \cap C)$ .

On the third hand, we could say "there are three pieces here" and describe the region as the union of the three pieces: It is  $(A' \cap (B \cap C')) \cup (A' \cap (B \cap C)) \cup (A' \cap (B' \cap C))$

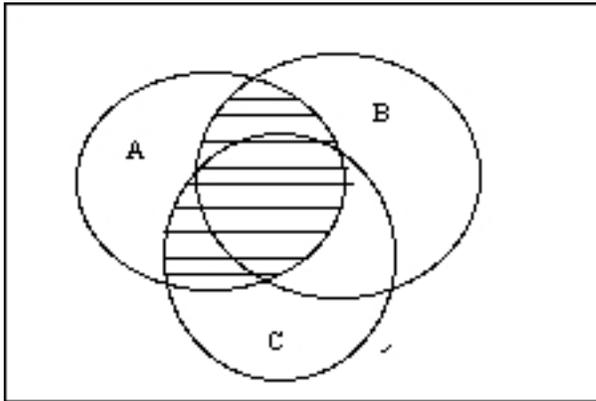
All three of these (and a few others) are correct descriptions of the shaded region - there can be many different descriptions of the same set, which means that there are a lot of rules for rewriting the description of a (complicated) set. This example gives one instance - showing that t it is always true that

$$(A' \cap B) \cup (A' \cap C) = (A' \cap B) \cup (A' \cap C) = (A' \cap (B \cap C')) \cup (A' \cap (B \cap C)) \cup (A' \cap (B' \cap C))$$

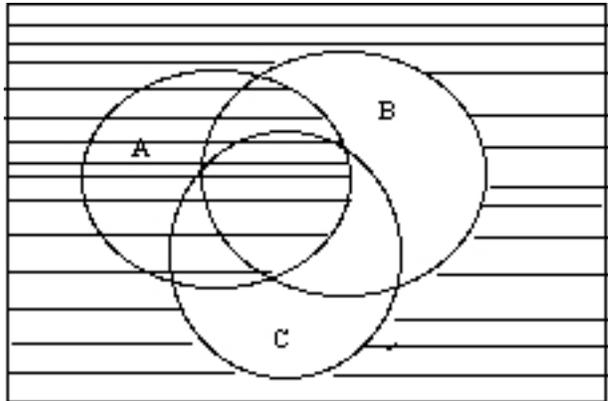
**EXERCISE**

- For each of these, draw a Venn diagram and mark the set that is described (you need three sets):
  - $A \cap (B' \cup C)$
  - $(A \cap B') \cup C$
  - $(A \cap B)' \cup C$
- For each of these, determine if the statement is always true by drawing (and shading) Venn diagrams of the two sets and comparing them:
  - $(A \cap B)' = A' \cap B'$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- Write the set description (using complements, unions, intersections as needed) for each of the sets shown here:

(a)



(b)



- Consider the following sets: The universal set is  $U = \{\text{College students}\}$ , and we have  $A = \{\text{Saint Mary's students}\}$ ,  $B = \{\text{Sophomores}\}$ ,  $C = \{\text{English Majors}\}$ 
  - Write the set notation (using unions, intersections, complements as appropriate) for the set of Saint Mary's sophomores who are not English Majors.
  - Write the set notation (using unions, intersections, complements as appropriate) for the set of Saint Mary's students who are sophomores or are English Majors.
  - Describe in words (as in (a) and (b)) the people in the set  $A' \cap (B \cup C)$

**READING ASSIGNMENT** (in preparation for next class meeting)

Reread Section 1.2 (as needed) and read Section 1.3 in the text

**SKILL EXERCISES:**(hand in - individually - at next class meeting)

p.19 # 8 - 10, 15 - 16