Analysis Lab 12

Topic: Uniform Convergence of a Sequence of Functions

Guidelines for Lab Report

For this lab, submit a report according to guidelines given below.

1. Complete Questions 1-3 in Section 2, and write your answers on pages 2-4 of this report guide.
2. Write your answers to Questions 1-3 of Section 3 on pages 5-8 of this report guide.
3. Write your answers to Questions 1-3 of Section 4 on page 9 of this report guide.
4. Complete the Questions for Reflection as assigned by your instructor. Write your response to each question on a separate sheet(s), and attach to the rest of this report.

2 Using Examples to Understand Pointwise Limits

Enter your responses below.

1. $(f_n(x) = x^n)_{n=1}^{\infty}$, $[0, 1]$

   (b) Write the first 6 terms of the sequence $(f_n(.5))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(.5)$?

   (c) Does the graph you created in Part (a) support your answer to Part (b)?

   (d) Compute the first 6 terms of the sequence $(f_n(.2))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(.2)$?

   (e) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(.2)$ is?

   (f) Write the first 6 terms of the sequence $(f_n(.7))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(.7)$?

   (g) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(.7)$ is?

   (h) For any $x_0 \neq 1$, what does it appear that $\lim_{n\to\infty} f_n(x_0)$ is?

   (i) What does it appear that $\lim_{n\to\infty} f_n(1)$ is?

   (j) What is the pointwise limit function f?

   (k) Graph the function f.

   (l) What can you say about the continuity of each f_n on the interval [0,1]?

   (m) What can you say about the continuity of f on the interval [0,1]?

2. $\left(f_n(x) = \dfrac{x^n}{1+x^n}\right)_{n=1}^{\infty}$, $[0,2]$

(b) Write the first 6 terms of the sequence $(f_n(.5))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(.5)$?

(c) Does the graph you created in Part (a) support your answer to Part (b)?

(d) Write the first 6 terms of the sequence $(f_n(.4))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(.4)$?

(e) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(.4)$ is?

(f) Write the first 6 terms of the sequence $(f_n(.1))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(.1)$?

(g) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(.1)$ is?

(h) Using the graph, for any $x_0 \in [0,1)$, what does it appear that $\lim_{n\to\infty} f_n(x_0)$ is?

(i) Write the first 6 terms of the sequence $(f_n(1))_{n=1}^{\infty}$. What does it appear that $\lim_{n\to\infty} f_n(1)$ is?

(j) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(1)$ is?

(k) Write the first 6 terms of the sequence $(f_n(1.2))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(1.2)$?

(l) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(1.2)$ is?

(m) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(1.4)$ is?

(n) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(1.8)$ is?

(o) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(2)$ is?

(p) Using the graph, for any $x_0 \in (1,2]$,what does it appear that $\lim_{n\to\infty} f_n(x_0)$ is?

(q) What is the pointwise limit f?

(r) Graph the function f.

(s) What can you say about the continuity of each f_n on the interval [0,2]?

(t) What can you say about the continuity of f on the interval [0,2]?

3. $\left(f_n(x) = \dfrac{x}{1+nx^2}\right)_{n=1}^{\infty}$, $[0,1]$

(b) Write the first 20 terms of the sequence $(f_n(.5))_{n=1}^{\infty}$. What is $\lim_{n\to\infty} f_n(.5)$?

(c) Does the graph you created in Part (a) support your answer to Part (b)?

(d) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(.2)$ is?

(e) Using the graph, what does it appear that $\lim_{n\to\infty} f_n(1)$ is?

(f) Using the graph, for any $x_0 \in [0,1]$, what does it appear that $\lim_{n\to\infty} f_n(x_0)$ is?

(g) What is the pointwise limit f?

(h) Graph the function f.

(i) What can you say about the continuity of each f_n on the interval [0,1]?

(j) What can you say about the continuity of f on the interval [0,1]?

3 Understanding the Two Types of Convergence

1. $(f_n(x) = x^n)_{n=1}^{\infty}$, $[0, 1]$

   (a) $\epsilon = .5$

      i. Does it appear that $f_{10}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

         If not, identify those points x for which $f_{10}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

      ii. Does it appear that $f_{15}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

         If not, identify those points x for which $f_{15}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

      iii. Explanation:

   (b) $\epsilon = .2$

      i. Does it appear that $f_{10}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

         If not, identify those points x for which $f_{10}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

      ii. Does it appear that $f_{15}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

         If not, identify those points x for which $f_{15}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

      iii. Explanation:

   (c) $\epsilon = .1$

      i. Does it appear that $f_{10}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

         If not, identify those points x for which $f_{10}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

ii. Does it appear that $f_{15}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

If not, identify those points x for which $f_{15}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

iii. Explanation:

2. $\left(f_n(x) = \dfrac{x^n}{1 + x^n}\right)_{n=1}^{\infty}$, $[0, 2]$

(a) $\epsilon = .3$

i. Does it appear that $f_{100}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [.95, 1.05]$?

If not, identify those points x for which $f_{100}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

ii. Does it appear that $f_{300}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [.95, 1.05]$?

If not, identify those points x for which $f_{300}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

iii. Explanation:

(b) $\epsilon = .2$

i. Does it appear that $f_{100}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [.95, 1.05]$?

If not, identify those points x for which $f_{100}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

ii. Does it appear that $f_{300}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [.95, 1.05]$?

If not, identify those points x for which $f_{300}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

iii. Explanation:

(c) $\epsilon = .1$

i. Does it appear that $f_{100}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [.95, 1.05]$?

If not, identify those points x for which $f_{100}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

ii. Does it appear that $f_{300}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [.95, 1.05]$?

If not, identify those points x for which $f_{300}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

iii. Explanation:

3. $\left(f_n(x) = \dfrac{x}{1 + nx^2}\right)_{n=1}^{\infty}$, $[0, 1]$

(a) $\epsilon = .3$

i. Does it appear that $f_{100}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

If not, identify those points x for which $f_{100}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

ii. Does it appear that $f_{300}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?
If not, identify those points x for which $f_{300}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

iii. Explanation:

(b) $\epsilon = .1$

i. Does it appear that $f_{100}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

If not, identify those points x for which $f_{100}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

ii. Does it appear that $f_{300}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?
If not, identify those points x for which $f_{300}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

iii. Explanation:

(c) $\epsilon = .05$

i. Does it appear that $f_{100}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?

If not, identify those points x for which $f_{100}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

ii. Does it appear that $f_{300}(x) \in (f(x) - \epsilon, f(x) + \epsilon)$ for all $x \in [0, 1]$?
If not, identify those points x for which $f_{300}(x) \notin (f(x) - \epsilon, f(x) + \epsilon)$.

iii. Explanation:

4 Critical Thinking Questions

1. Examining the results of the last section, explain in your own words the difference between the behavior of the sequences from Questions 1 and 2 versus the sequence from Question 3.

2. N

3. $(f_n(x) = nx\mathrm{e}^{-n^2 x})_{n=1}^{\infty}$, $[0, 1]$

   (b) What is the pointwise limit f?

   (d) N

   (e) What can you say about the continuity of each f_n on the interval [0,1]?

   (f) What can you say about the continuity of f on the interval [0,1]?