# Analysis Lab 6 

## Topic: Algebraic Combinations of Sequences

## Guidelines for Lab Report

For this lab, submit a report according to guidelines given below.

1. Complete the table from Section 2.1 that is provided on the following page. Submit your statement of Conjecture 1, as well as your responses to Questions 1 and 2 from Section 2.3.
2. Complete the table from Section 3.1 that is provided on page 4 of this report guide. Submit your statement of Conjecture 2, as well as your responses to Questions 1 and 2 from Section 3.3.
3. Complete the table from Section 4.1 that is provided on page 6 of this report guide. Submit your statement of Conjecture 3, as well as your responses to Questions 1 and 2 from Section 3.3.
4. Complete the Questions for Reflection as assigned by your instructor. Write your response for each question on a separate sheet(s), and attach to the rest of this report.

## 2 The Sum of Two Convergent Sequences

### 2.1 Formulating a Conjecture

| Set | $\lim _{n \rightarrow \infty} a_{n}$ | $\lim _{n \rightarrow \infty} b_{n}$ | $\left(a_{n}+b_{n}\right)$ | $\lim _{n \rightarrow \infty}\left(a_{n}+b_{n}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| Set I $\begin{gathered} \left(a_{n}\right)_{n=1}^{\infty}=\left(\frac{1}{n}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left(\frac{5 n-2}{n+4}\right)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |
| Set II $\begin{aligned} \left(a_{n}\right)_{n=1}^{\infty} & =\left(\frac{1-2 n}{n+1}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty} & =\left(2-\frac{1}{n^{2}}\right)_{n=1}^{\infty} \end{aligned}$ |  |  |  |  |
| $\begin{gathered} \text { Set III } \\ \left(a_{n}\right)_{n=2}^{\infty}=\left(\sin \left(\frac{n \pi}{2}\right)\right)_{n=2}^{\infty} \\ \left(b_{n}\right)_{n=2}^{\infty}=\left(\frac{1}{\ln n}\right)_{n=2}^{\infty} \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { Set IV } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(1+\frac{2}{n}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left(\left\{\begin{array}{cc} 3-1 / n, & \text { if } n \text { is even } \\ 3, & \text { if } n \text { is odd } \end{array}\right)_{n=1}^{\infty}\right. \end{gathered}$ |  |  |  |  |
| Set V $\begin{gathered} \left(a_{n}\right)_{n=1}^{\infty}=\left((-1)^{n}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left((-1)^{n+1}\right)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { Set VI } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(1^{n}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left((-1)^{n}\right)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |

## Conjecture 1:

### 2.3 The Conjecture and Its Proof

In the space below, provide your responses to Questions 1 and 2. Attach additional sheet(s), if necessary.

## 3 The Product of Two Convergent Sequences

### 3.1 Formulating a Conjecture

| Set | $\lim _{n \rightarrow \infty} a_{n}$ | $\lim _{n \rightarrow \infty} b_{n}$ | $\left(a_{n} \cdot b_{n}\right)$ | $\lim _{n \rightarrow \infty}\left(a_{n} \cdot b_{n}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Set I } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(\frac{2+3 n}{n+4}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left(\frac{5 n-2}{n+4}\right)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { Set II } \\ \left(a_{n}\right)_{n=1}^{\infty}=(n)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left(\frac{1}{n^{2}}\right)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |
| Set III $\begin{aligned} & \left(a_{n}\right)_{n=1}^{\infty}=\left(3^{\frac{1}{n}}\right)_{n=1}^{\infty} \\ & \left(b_{n}\right)_{n=1}^{\infty}=\left(2^{\frac{1}{n^{2}}}\right)_{n=1}^{\infty} \end{aligned}$ |  |  |  |  |
| $\begin{gathered} \text { Set IV } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(3+\frac{1}{n}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left(\left\{\begin{array}{cc} 2-\frac{1}{n^{2}}, & \text { if } n \text { is even } \\ 2, & \text { if } n \text { is odd } \end{array}\right)_{n=1}^{\infty}\right. \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { Set V } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(\sin \left(\frac{n \pi}{2}\right)\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left((-1)^{n}\right)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |

## Conjecture 2:

### 3.3 The Conjecture and Its Proof

In the space below, provide your responses to Questions 1 and 2. Attach additional sheet(s), if necessary.

## 4 The Quotient of Two Convergent Sequences

### 4.1 Formulating a Conjecture

| Sequence | $\lim _{n \rightarrow \infty} a_{n}$ | $\left(1 / a_{n}\right)$ | $\lim _{n \rightarrow \infty}\left(1 / a_{n}\right)$ |
| :---: | :---: | :---: | :---: |
| $\left(3-\frac{2}{n}\right)_{n=1}^{\infty}$ |  |  |  |
| $\left(\sin \left(\frac{n \pi}{2}\right)\right)_{n=1}^{\infty}$ |  |  |  |
| $\left(\frac{2 n}{3 n+4}\right)_{n=1}^{\infty}$ |  |  |  |
| $\left(\frac{1}{n^{2}}\right)_{n=1}^{\infty}$ |  |  |  |
| $\left(\left\{\begin{array}{c}3-\frac{1}{n}, \text { if } n \text { is even } \\ \text { if } n \text { is odd }\end{array}\right)_{n=1}^{\infty}\right.$ |  |  |  |
| $\left(\frac{n!}{25^{n}}\right)_{n=1}^{\infty}$ |  |  |  |

Conjecture 3:

### 4.2 The Conjecture and Associated Proofs

In the space below, provide your responses to Questions 1-3. Attach additional sheet(s), if necessary.

## 5 Weakening the Condition Involving Products

| Set | $\lim _{n \rightarrow \infty} a_{n}$ | $\lim _{n \rightarrow \infty} b_{n}$ | $c_{n}=a_{n} b_{n}$ | $\lim _{n \rightarrow \infty} c_{n}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Set I } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(\frac{1}{n}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left((-1)^{n}\right)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { Set II } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(\frac{1}{n^{2}}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=(\sin n)_{n=1}^{\infty} \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { Set III } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(\frac{4}{n^{3}}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left(\left\{\begin{array}{cc} 2-1 / n, & \text { if } n \text { is even } \\ -2+1 / n, & \text { if } n \text { is odd } \end{array}\right)_{n=1}^{\infty}\right. \end{gathered}$ |  |  |  |  |
| $\begin{gathered} \text { Set IV } \\ \left(a_{n}\right)_{n=1}^{\infty}=\left(\frac{(-1)^{n}}{n}\right)_{n=1}^{\infty} \\ \left(b_{n}\right)_{n=1}^{\infty}=\left(\left\{\begin{array}{cc} 2, & \text { if } n \text { is even } \\ 1 / n, & \text { if } n \text { is odd } \end{array}\right)_{n=1}^{\infty}\right. \end{gathered}$ |  |  |  |  |

In the space below, provide your responses to Questions 1-5. Attach additional sheet(s), if necessary.

